Math02B Assignment 7.1 Last Name_______________1st_________

Give complete solutions to the following problems. Be sure to provide all the necessary steps to support your answers.

1. Determine which of the matrices below are orthogonal. If orthogonal, find the inverse.

a. $\begin{bmatrix} 0.3 & 0.4 \\ 0.4 & -0.3 \end{bmatrix}$

b. $\begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}$

c. $\begin{bmatrix} -1 & 2 & 0 \\ 2 & -1 & 2 \\ 0 & 2 & -1 \end{bmatrix}$

d. $\begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}$

2. Orthogonally diagonalize the given matrices, giving an orthogonal matrix P and a diagonal matrix D

a. $\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 1 & 5 \\ 1 & 5 & 1 \\ 5 & 1 & 1 \end{bmatrix}$. $\lambda = -4, 4, 7$

3. Let U be a unit vector in $\Re^n$, and let B = **u u**^{T}.

a. Given any **x** in $\Re^n$, compute B**x** and show that B**x** is the orthogonal projection of **x** onto **u**, as described in Section 6.2.

b. Show that B is a symmetric matrix and $B^2 = B$.

c. Show that **u** is an eigenvector of B. What is the corresponding eigenvalue?

4. Let B be an nxn symmetric matrix such that $B^2 = B$. Any such matrix is called a Projectin Matrix (or an orthogonal projection matrix) Given any y in $\Re^n$, let $\hat{\mathbf{y}} = \mathbf{By}$ and $z = y - \hat{y}$.

a. Show that **z** is orthogonal to $\hat{y}$.

b. Show that **y** is the sum of a vector in the column space of **B** and a vector in the orthogonal complement of the column space of **B**. State why that proves **By** is the orthogonal projection of **y** onto the column space of **B**.