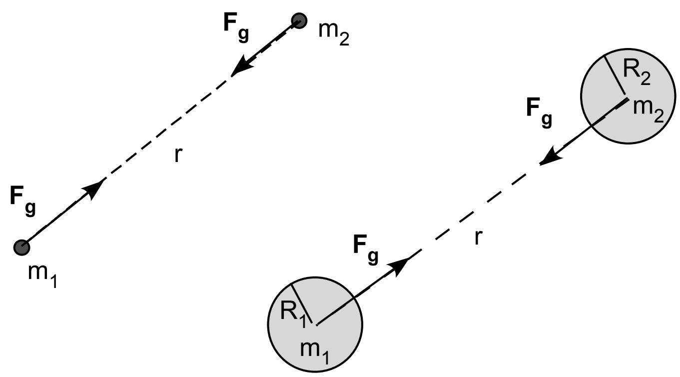
Law of Gravitation

In 1687 Isaac Newton published his Law of Gravitation which is stated mathematically in the following form:

$$\boxed{F_g = \frac{Gm_1m_2}{r^2}}$$ Law of Gravitation

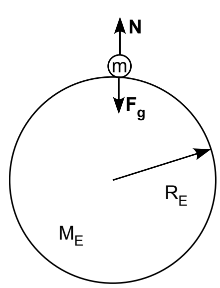
$$G = 6.672\times10^{-11}\frac{Nm^2}{kg^2}$$ (Gravitation Constant)



Although the Law of Gravity applies to particles (point masses), Newton also showed that it applies to spherically symmetric bodies because they behave as if their mass was concentrated at their center. If the bodies are not symmetrical, you can still compute the gravitational force between them by using the distance between the center of mass of both objects.

Weight

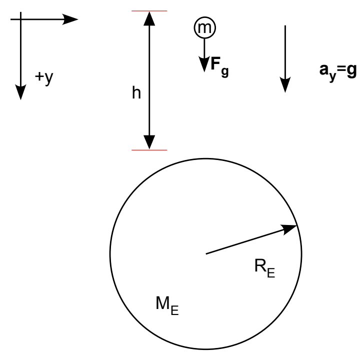
Consider an object at rest on earth' surface:



$$\boxed{W = F_g = \frac{GM_Em}{R_E^2}}$$ Weight of an object near earth's surface

Acceleration of Gravity

Consider an object of mass 'm' in free-fall at a height 'h' from earth's surface:



$$\sum F_y = F_g = ma_y$$

$$\frac{GM_E m}{(R_E + h)^2} = mg$$

$$\boxed{g = \frac{GM_E}{(R_E + h)^2}}$$ Acceleration of Gravity

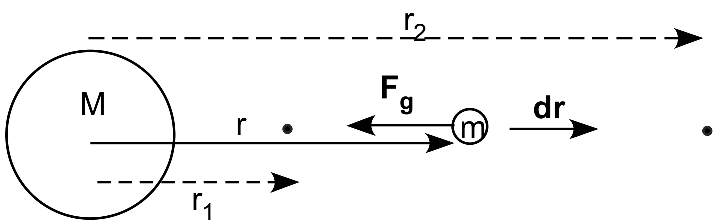
At surface of earth h = 0:

$$g = \frac{GM_E}{R_E^2} = \frac{\left(6.672 \times 10^{-11} \frac{Nm^2}{kg^2}\right)\left(5.97 \times 10^{24} kg\right)}{\left(6.38 \times 10^6 m\right)^2} \approx 9.8 m/s^2$$

At h = 400 km (Shuttle's Orbit):

$$g = \frac{GM_E}{R_E^2} = \frac{\left(6.672 \times 10^{-11} \frac{Nm^2}{kg^2}\right)\left(5.97 \times 10^{24} kg\right)}{\left(6.38 \times 10^6 m + 400 \times 10^3 m\right)^2} = 8.7 m/s^2$$

Gravitational Potential Energy
Consider a planet of mass *M* and a particle of mass *m*. Let's find the work done by gravity in moving the particle from r_1 to r_2 along a straight path.



$$w_g = \int_{r_1}^{r_2} \vec{F} \bullet d\vec{l} = \int_{r_1}^{r_2} \vec{F} \bullet d\vec{r} = \int_{r_1}^{r_2} -\frac{GMm}{r^2} dr$$

$$w_g = -GMm \int_{r_1}^{r_2} \frac{1}{r^2} dr = -GMm \left(-\frac{1}{r} \right)_{r_1}^{r_2}$$

$$w_g = \frac{GMm}{r_2} - \frac{GMm}{r_1}$$

Since $\mathbf{F_g}$ is conservative:

$$w_g = -\Delta U_g$$
$$\Delta U_g = -w_g$$
$$U_2 - U_1 = \frac{GMm}{r_1} - \frac{GMm}{r_2}$$

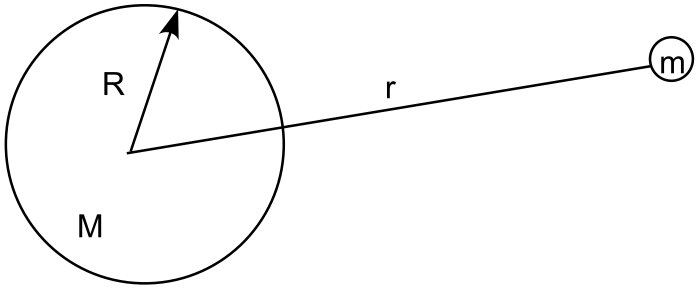
Since the reference point for the PE function is completely arbitrary, it is convenient to define the PE to be zero when the force on the particle is also zero. Thus, we will take:

$$U_2 = 0 \text{ when } r \rightarrow \infty$$

Therefore,

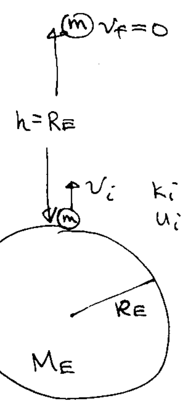
$$U_2(\infty) - U_1 = \frac{GMm}{r_1} - \lim_{r_2 \rightarrow \infty} \frac{GMm}{r_2}$$

$$\boxed{U(r) = -\frac{GMm}{r}}$$ Gravitational PE Function



$$U(r) = -\frac{GMm}{r} \quad \text{valid for } r \geq R$$

Ex. a) Calculate the initial vertical launch speed of a projectile required to reach a height above the earth equal to the earth's radius.



$K_f = 0$

$U_f = -\frac{GMm}{R_E + h}$

$K_i = \frac{1}{2} m v_i^2$

$U_i = -\frac{GMm}{R_E}$

$$K_i + U_i = K_f + U_f \quad (K_f \to 0)$$

$$\frac{1}{2} m v_i^2 - \frac{GMm}{R_E} = -\frac{GMm}{2R_E}$$

$$\frac{1}{2} m v_i^2 = \frac{1}{2}\frac{GMm}{R_E}$$

$$v_i = \sqrt{\frac{GM_E}{R_E}}$$

$$v_i = \sqrt{\frac{(6.672\times10^{-11})(5.97\times10^{24})}{6.38\times10^{6}\ \text{m}}}$$

$$v_i = 7900\ \text{m/s} \approx 17{,}700\ \text{mp}$$

b) Calculate the minimum speed so that the projectile escapes from the earth's gravitational effect.

$$K_i + U_i = K_f + U_f \quad (K_f \to 0,\ U_f \to 0)$$

$$\frac{1}{2} m v_{esc}^2 - \frac{GMm}{R_E} = 0$$

$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}} = 1.12\times10^{4}\ \text{m/s} \approx 25{,}000\ \text{mph}$$

$$\boxed{v_{esc} = \sqrt{\frac{2GM}{R}}}$$

Escape Speed