Matter Waves

To form a wave-packet (matter wave) that is finite over a limited range Δx and zero everywhere else requires that you add an infinite number of harmonic waves with continuously different frequency (f), wavelength (λ), and amplitude. This is obtained by using a Fourier Integral which is defined as:

$$f(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} a(k) e^{i(kx-\omega t)} dk$$

a(k) = amplitude distribution function

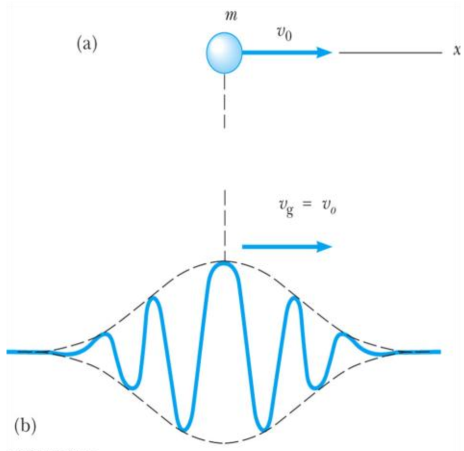
$$e^{i(kx-\omega t)} = \cos(kx - \omega t) + i \sin(kx - \omega t)$$

This Fourier Integral is analogous to the Fourier Series:

$$f(x,t) = \sum_{-\infty}^{+\infty} A_i \cos(k_i x - \omega_i t)$$

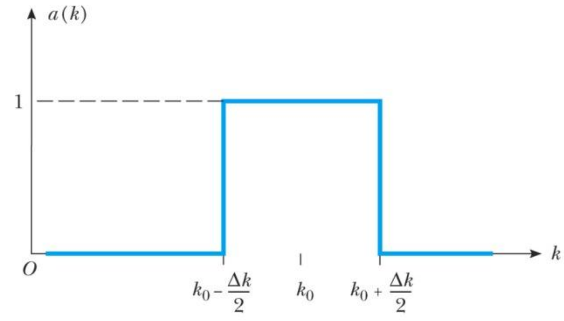
A_i = Fourier Coefficients

The resulting matter wave packet and then be used to describe the motion of a particle!!!!



Ex. Matter Wave Packet

Given *a(k) = 1* (see figure below) find the Matter Wave Packet function.



$$f(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} a(k) e^{i(kx-\omega t)} dk$$

$$f(x,t) = \frac{1}{\sqrt{2\pi}} e^{-i\omega t} \int_{k_o-\Delta k/2}^{k_o+\Delta k/2} (1) e^{ikx} dk$$

$$f(x,t) = \frac{1}{\sqrt{2\pi}} e^{-i\omega t} \int_{k_o-\Delta k/2}^{k_o+\Delta k/2} \cos(kx) + i\sin(kx)\ dk$$

$$f(x,t) = \frac{1}{\sqrt{2\pi}} \frac{\sin\left(\Delta k \frac{x}{2}\right)}{\left(\Delta k \frac{x}{2}\right)} e^{i(k_o x-\omega t)}$$

using, $e^{i(kx_o-\omega t)} = \cos(k_o x - \omega t) + i\sin(k_o x - \omega t)$

$$f(x,t) = \frac{1}{\sqrt{2\pi}} \frac{\sin\left(\Delta k \frac{x}{2}\right)}{\left(\Delta k \frac{x}{2}\right)} \left[\cos(k_o x - \omega t) + i\sin(k_o x - \omega t)\right]$$