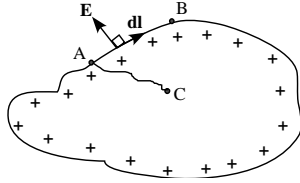


Potential of a Charged Conductor

We will now prove that the surface of a charged conductor in electrostatic equilibrium is an equipotential surface - the surface is at the same electric potential.

Consider a charged conductor in electrostatic equilibrium.



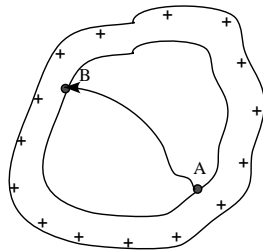
$$V_B - V_A = - \int_A^B \vec{E} \cdot d\vec{\ell} = 0 \quad (\text{since } \vec{E} \perp d\vec{\ell} \text{ between A and B})$$

$$V_C - V_A = - \int_A^C \vec{E} \cdot d\vec{\ell} = 0 \quad (\text{since } E=0 \text{ everywhere inside a conductor})$$

1. The surface of a charged conductor in equilibrium is an equipotential surface.
2. Since $E = 0$ inside the conductor, then the potential must be constant everywhere inside the conductor and equal to the value at the surface

A Cavity Within a Conductor

We will now prove that if you have a cavity inside a conductor with no charge inside, the electric field must be zero everywhere inside the cavity.



$$V_B - V_A = - \int_A^B \vec{E} \cdot d\vec{\ell} = 0$$

Since every point on the conductor must be at the same potential. This must be true for every path between A and B. The only way this can be true is if $E = 0$ everywhere inside the cavity. Since $E = \sigma / \epsilon_0$ at the surface of a conductor, then $\sigma = 0$, and thus there cannot be any charge on the surface of the cavity. Thus, if you're inside a charged conducting box, you can safely touch any point inside the wall of the box safely without getting shocked.

If you have a cavity inside of a conductor and no charge inside the cavity:

1. $E = 0$ everywhere inside the cavity.
2. Since $E = \sigma / \epsilon_0 = 0$ at the surface of a conductor, then $\sigma = 0$ and thus there cannot be any charge on the surface of the cavity.